

$$P(B|A)$$

2nd 1st

$$P(A|B)$$

1st 2nd

AP Stat- Ch. 15 (part 2) **TREE DIAGRAMS (Bayes' Rule)**

EXAMPLE: An airline offers discounted tickets to customers who buy tickets early (more than 30 days ahead of time). The company has noticed that 60% of its customers take advantage of the "early-bird" fares, while 25% purchase regular fares, and 15% are "last-minute" customers (less than 48 hours before the flight). They have also figured out that the no-show rate among "early-bird" purchasers is 30%, and only 8% among regular fare customers, and 2% among last-minute customers.

Create a tree diagram of this situation:

$$P(E) = 0.60$$

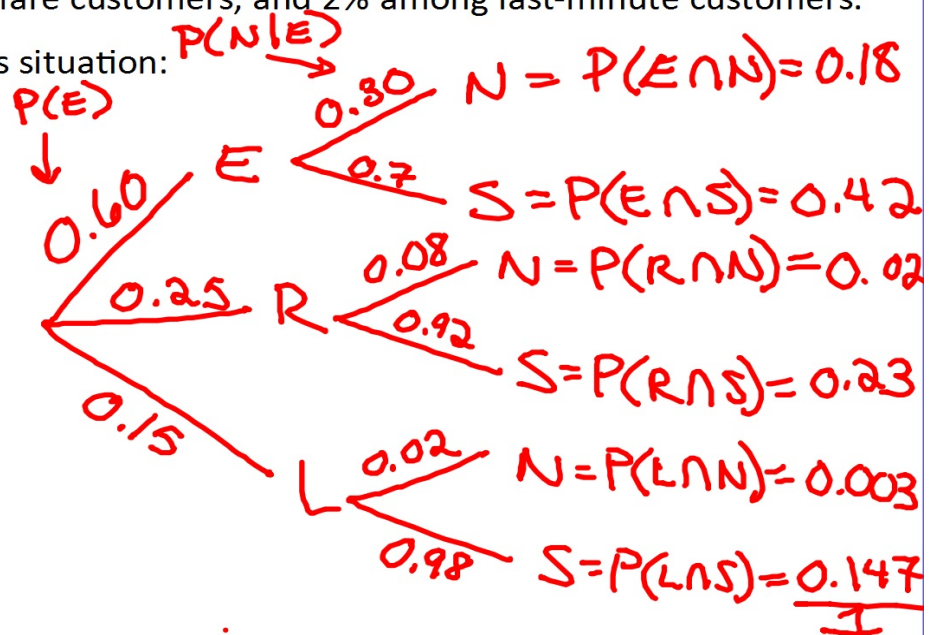
$$P(R) = 0.25$$

$$P(L) = 0.15$$

$$P(N|E) = 0.30$$

$$P(N|R) = 0.08$$

$$P(N|L) = 0.02$$



So now let's answer some easy questions:

1) What is the probability of being a no-show (overall)?

$$P(N) = 0.203$$

2) What is the probability of being a no-show AND a last minute customer?

$$P(N \cap L) = 0.003$$

3) What is the probability of being a no-show AND an early-bird customer?

$$P(N \cap E) = 0.18$$

Now let's try some harder ones:

1. Given that you have a no-show in a certain seat, what is the probability of that person being a last-minute customer?

$$P(L|N) = \frac{P(L \cap N)}{P(N)} = \frac{0.003}{0.203} = 0.0148$$

2. Given that you have a no-show in a certain seat, what is the probability of that person being a regular-fare customer?

$$P(R|N) = \frac{P(R \cap N)}{P(N)} = \frac{0.02}{0.203} = 0.0985$$

3. Given that you have a person show up for a certain seat, what is the probability of that person being an early-bird customer?

$$P(E|S) = \frac{P(E \cap S)}{P(S)} = \frac{0.42}{0.797} = 0.527$$

Example #1

- A cancer clinic gives free cancer test (*hypothetically*)
- It is known that 2% of the people that come into the clinic have cancer (*hypothetically*)
- It is known the test comes up positive in 98% of people with cancer
- It is known the test comes up positive in 3% of people without cancer

Create the tree diagram:

~~1st event =~~

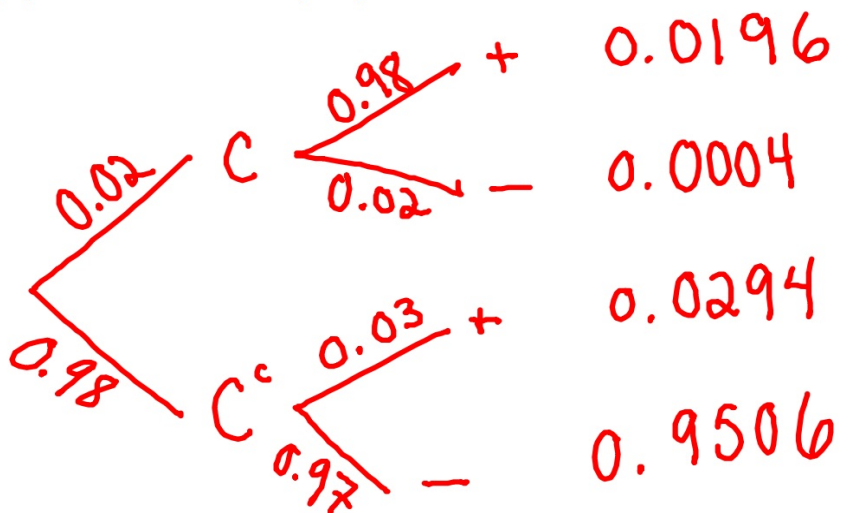
~~2nd event =~~

$$P(C) = 0.02$$

$$P(C^c) = 0.98$$

$$P(+|C) = 0.98$$

$$P(+|C^c) = 0.03$$



Answer the following questions:

1- What is the probability that someone tests positive **given that** they have cancer?

$$P(+|C) = 0.98$$

2- What is the probability that someone tests positive **given that** they don't have cancer?

$$P(+|C^c) = 0.03$$

3- What is the probability that someone tests negative **given that** they have cancer?

$$P(-|C) = 0.02$$

4- What is the probability that someone tests negative **given that** they don't have cancer?

$$P(-|C^c) = 0.97$$

5- What is the probability that someone tests positive? Negative?

$$P(+) = 0.049 \quad P(-) = 0.951$$

6- What is the probability that someone has cancer **given that** they test positive?

(This is called the accuracy of the test)

$$P(C|+) = \frac{0.0196}{0.049} = 0.40$$

7- What is the probability that someone doesn't have cancer **given that** they test positive?

(this is called a false positive)

$$P(C^c|+) = 0.60$$

8- What is the probability that someone has cancer **given that** they test negative?

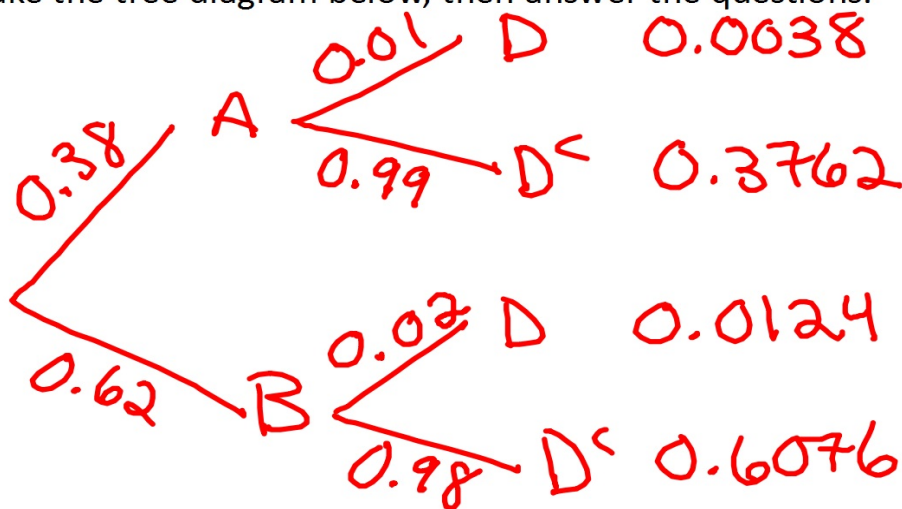
$$\begin{aligned} P(C|-) &= 0.000421 \\ &= \frac{0.0004}{0.951} \end{aligned}$$

Example #2:

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

$$P(D|A) = 0.01 \quad P(D|B) = 0.02 \quad P(A) = 0.38$$
$$P(B) = 0.62$$

Make the tree diagram below, then answer the questions:



Questions:

1- What is the probability that a book is NOT defective?

$$P(D^c) = 0.9838$$

2- If a book is not defective, what's the probability that it came from company B?

$$P(B | D^c) = \frac{0.6076}{0.9838} = 0.6176$$

3- If we open a book and it **IS** defective, what's the probability that is from company

A? Company B?

$$P(D) = 0.0162$$

$$P(A|D) = \frac{0.0038}{0.0162} = \underline{\underline{0.235}}$$

$$P(B|D) = \frac{0.0124}{0.0162} = \underline{\underline{0.7654}}$$

TRY THE TWO PROBLEMS ON THE NEXT PAGE
and THE WORKSHEET AFTER THAT